Simple Scaling Parameters for Air-Breathing Engines

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Nomenclature

a,b,c	= see Eqs. (7) and (8)
A	= const of proportionality, Eq. (4)
B	= conversion factor from heat to mechanical units
D	= see Eqs. (7) and (8)
e	= see Eqs. (7) and (8)
E	= energy in the propulsive fluid
$(E)_{1,2,\ldots,6}$	= see Fig. 1
I	= mechanical inertia of rotating parts
N	= rotational speed
Q	= energy input rate (fuel)
t	= time
x	= ΔN
δ	= delay
δ_α	= aerodynamic delay
δ_f	= combustion or energy input delay

IN the development of facilities for the testing of air-breathing engines, it is well known that the results obtained from a model of an engine component or of a complete engine will apply to the full scale machine only in selected ranges of operation. Such ranges of operation are, in general, not amenable to being identified by scaling parameters, and a very complicated program of integrated tests or very extensive analog-digital computer solutions will become necessary before different engines can be compared in the steady and the transient states. Although recognizing the large number of variables influencing the operation of an air-breathing engine in an aircraft, the generally random nature of the inputs to the system, and the multiply coupled nonlinear nature of any analytical model that may be attempted for representing the engine, it appears that a simple problem may be set as follows to determine the influence of size reduction on the performance of the scaled engine as compared with the performance of the original engine: given an air-breathing engine for assessment or testing under nonsteady-state operating conditions, what design and operational parameters will indicate the possible differences in the performance of a scaled engine as compared with the performance of the original engine?

A simple engine configuration is selected as shown in Fig. 1. At the outset, it must be recognized that the engine is being considered as a system in which the inlet and the outlet conditions are those obtained from the in-flight aircraft configuration. Neither the propulsive efficiency nor the engine thermodynamic efficiency are under consideration. The original engine is to be scaled to a smaller size and the transient or the nonsteady performance of each is examined.

In the absence of a frictional force, it is clear that a simple energy balance equation may be written as follows for the configuration being considered:

steady state:
$$E_6 - E_1 = BQ \tag{1}$$

nonsteady state:
$$I(dN/dt) = BQ(t - \delta_f) + E_1(- \delta_1) - E_6(- \delta_2) \tag{2}$$

where δ_f, δ_1, and δ_2 are the delay times involved in the combustion chamber, the aerodynamic inlet and compressor, and the turbine and the nozzle. Assuming for the purposes of this analysis that all of the delays associated with aerodynamic inertia can be added together to an amount δ_α, one may rewrite Eq. (2) as follows:

$$I(dN/dt) = BQ(t - \delta_f) + (E_1 - E_6)(t - \delta_\alpha) \tag{3}$$

In studies on steady-state engine cycle analysis, it is found that a relation can generally be established between E_1, E_6, and $(E_1 - E_6)$ and N, the rotational speed of the engine. A relation which is applicable in many practical systems is given by (see Ref. 1)

$$(E_1 - E_6)(t - \delta_\alpha) = -AN^2(t - \delta_\alpha) \tag{4}$$

where A is a constant of proportionality. It follows then from Eq. (3) that

$$I(dN/dt) = BQ(t - \delta_f) - AN^2(t - \delta_\alpha) \tag{5}$$

One can then consider a small variation 1) directly in fuel input or 2) indirectly in an equivalent fuel input, the equivalence, for example, being established from changed inlet conditions and consequentially the new energy release in the combustion chamber. If then, Δ represents the difference between the instantaneous and the steady-state values, Eq. (5) becomes

$$I(d/dt)(\Delta N) = B\cdot\Delta Q(t - \delta_f) - 2AN(t - \delta_\alpha)\cdot\Delta N(t - \delta_\alpha) \tag{6}$$

Expanding $\Delta N(t - \delta_\alpha)$ in Taylor series and after some algebra, it follows, on writing $N = N_0 + \Delta N$, N_0 being the datum speed, that

$$\frac{d^2\Delta N}{dt^2} + \frac{I}{AN_0\delta_\alpha{}^2} - \frac{2}{\delta_\alpha} - \frac{I}{AN_0\delta_\alpha}\cdot\frac{\Delta N}{N_0}\,\frac{d}{dt}\cdot\Delta N + \frac{2}{\delta_\alpha{}^2}\cdot\Delta N = \frac{\Delta Q(t - \delta_f)}{AN^2\delta_\alpha{}^2}\left(1 - \frac{\Delta N}{N_0}\right) \tag{7}$$

which is of the form

$$d^2x/dt^2 + (a - bx)(dx/dt) + cx = D(1 - ex) \tag{8}$$

This is the governing equation of the simple system considered in Fig. 1. It is an ordinary nonlinear differential equation well known in literature. This equation can be examined under three cases:

Case 1: $a > 0$. The equilibrium point is given by

$$x_s = D/(c + De) \qquad y_s = 0$$

One can then distinguish between the cases: a) $a < bx_s$ and $(a - bx_s)^2/4 > (c + De)$, unstable nodal point; b) $a < bx_s$ and $(a - bx_s)/4 < (c + De)$, unstable focal point; c) $a > bx_s$ and $(a - bx_s)/4 > (c + De)$, stable nodal point; d) $a > bx_s$ and $(a - bx_s)/4 < (c + De)$, stable focal point.

Case 2: $a < 0$. There arise small self-sustained oscillations.

Case 3: $a = 0$; a) $b < 0$: equilibrium position is unstable; b) $b > 0$: system performs simple harmonic oscillations, but trivial solution.

It is now possible to compare the performance of the original engine and of the scaled engine. Even in this simple con-

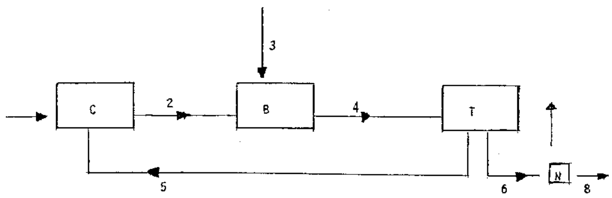

Fig. 1 Simple engine configuration. *C* is the compressor; *B*, the burner; *T*, the turbine; *N*, the nozzle; ( )₁, the inlet; ( )₃, the fuel energy added; ( )₅, the compressor work input; ( )₆, the energy available for useful work output; ( )₈, the energy let out unused.

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figuration, it is clear that

1) $I/AN_0\delta_\alpha - 2 \equiv a\delta_\alpha$ 2) $I/AN_0\delta_\alpha \equiv b$

3) $2/\delta_\alpha^2 \equiv c$ 4) $(t - \delta_f)/AN_0^2\delta_\alpha^2 \equiv D$

have to be individually and severally the same in the original and the model configurations. For example, the original engine may have a stable nodal point characteristic whereas the scaled engine may display small self-sustained oscillations if the changes in the inertias and the delays are not matched as required while an independent criterion determines the choice of rotational speed.

In general the model may have to be run at a different speed. The matching with respect to each of the parameters is essential in order to maintain similarity under dynamic conditions. The size, weight, rotational speed, and the delay times should, therefore, be so chosen in the model that such matching is assured and it is on this basis that the size of a test facility has to be considered for a given size of engine.

A more extensive analysis taking account of transients in individual components of the engine, and which is applicable to mixed cycles will be communicated elsewhere.

Reference

[1] Zucrow, M. J. and Murthy, S. N. B., "Jet propulsion and aircraft propellers," *Mark's Mechanical Engineer's Handbook* (McGraw-Hill Book Company Inc., New York, 1967), 7th ed.

Two-Dimensional Air-Cushion Vehicle Critical Forward Speeds

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Nomenclature

h = vehicle clearance (see Fig. 2)
H = jet total pressure (gage)
p_c = cushion pressure (gage)
M_T = total momentum flux from front and rear jets, per unit nozzle length for peripheral jet
t = jet thickness at nozzle exit (see Fig. 2)
t_c = jet thickness at station c (see Fig. 2)
V = mean jet velocity at nozzle exit [defined in Eq. (1)]
V_c = mean jet velocity at station c [defined in Eq. (2)]
V_f = critical forward speed for peripheral jet
θ = jet angle at nozzle exit (see Fig. 2)
ρ = air density
ψ = ratio of peripheral jet to plenum chamber critical forward speeds

Introduction

In recent years, a number of high-speed, guided land transport vehicles employing air-cushion support have been proposed, i.e., the Foa "Project Tubeflight,"[1] the Hovercraft Development Limited "Tracked Hovercraft,"[2] and the Bertin "Aerotrain."[3] The advantage of such support systems compared with conventional wheels is that the reduced "foot" pressures allow lower track construction and maintenance costs. Additionally, these systems remove frictional resistance and provide the vehicle with a suspension system, either completely or in part.

Two practical types of air-cushion suspension have been suggested—the plenum chamber and the peripheral jet. In the plenum chamber, air is pumped into a cavity formed

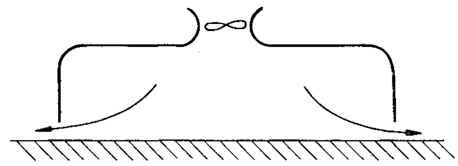

Fig. 1a Plenum chamber.

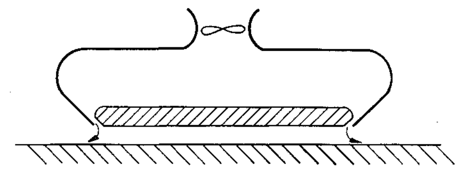

Fig. 1b Peripheral jet.

between the base of the vehicle and the track, and escapes around the periphery of the pad (Fig. 1a). The peripheral jet concept employs an air curtain to form a barrier across the gap between the track and the vehicle to impede the air escaping from the cushion (Fig. 1b). As it is proposed to support high-speed vehicles by these two systems, it is of interest to compare their sensitivity to the effect of forward speed. This can be done in a gross way by determining the vehicle speed at which air from the cushion is just prevented from escaping forward at the front of the support pad. The present note compares this "critical" velocity for two-dimensional peripheral jet and plenum chamber configurations. The justification for comparing the systems on a two-dimensional, rather than a three-dimensional basis is that the support pad configurations are usually either of high aspect ratio or effectively two-dimensional.

The prediction of the critical velocity for a two-dimensional peripheral jet was first determined in Ref. 4 by the relationship $\frac{1}{2}\rho V_f^2 = M_T/h$ where M_T had to be determined semi-empirically. This critical velocity can now be calculated using the simple forward speed theory of Alexander.[5]

Analysis

The magnitudes of the pressures and airflow velocities practically encountered are sufficiently small to allow the analysis to be reasonably performed treating the airflow as incompressible. Consider the front curtain of a two-dimensional peripheral jet vehicle travelling at the critical velocity, (Fig. 2). Following West[4] and Alexander,[5] it will be assumed that the static pressure on the upstream face of the air curtain is equal to the free-stream dynamic pressure. It will further be assumed that the static pressure at the nozzle plane is the average of the static pressure on either side of the air curtain. The validity of the latter assumption is examined in the discussion.

Therefore, applying Bernoulli's equation,

$$H = \tfrac{1}{2}\rho V^2 + \tfrac{1}{2}(p_c + \tfrac{1}{2}\rho V_f^2) \tag{1}$$

From the continuity equation

$$Vt = V_c t_c \tag{2}$$

and assuming that there are no flow energy losses between the nozzle exit and c

$$H = \tfrac{1}{2}\rho V_c^2 + p_c \tag{3}$$

From (1, 2, and 3)

$$\rho V_c^2 t_c = 2Ht\left\{\left[1 - \left(\frac{p_c}{H}\right)\right]\left[1 - \left(\frac{p_c + \frac{1}{2}\rho V_f^2}{2H}\right)\right]\right\}^{1/2} \tag{4}$$

The foregoing derivation follows that of the theory of Ref. 6, which is in reasonable agreement with experimental results.

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